Problem 11

In each of Problems 7 through 14, verify that each given function is a solution of the differential equation.

$$2t^2y'' + 3ty' - y = 0, \quad t > 0; \qquad y_1(t) = t^{1/2}, \quad y_2(t) = t^{-1}$$

Solution

$$\begin{aligned}
2t^2y_1'' + 3ty_1' - y_1 &\stackrel{?}{=} 0 \\
2t^2\frac{d^2}{dt^2}(t^{1/2}) + 3t\frac{d}{dt}(t^{1/2}) - t^{1/2} &\stackrel{?}{=} 0 \\
2t^2\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)t^{-3/2} + 3t\left(\frac{1}{2}\right)t^{-1/2} - t^{1/2} &\stackrel{?}{=} 0 \\
-\frac{1}{2}t^{1/2} + \frac{3}{2}t^{1/2} - t^{1/2} &\stackrel{?}{=} 0 \\
0 &= 0
\end{aligned}$$

The first solution is verified.

$$\begin{aligned}
2t^2y_2'' + 3ty_2' - y_2 &\stackrel{?}{=} 0 \\
2t^2\frac{d^2}{dt^2}(t^{-1}) + 3t\frac{d}{dt}(t^{-1}) - t^{-1} &\stackrel{?}{=} 0 \\
2t^2(-1)(-2)t^{-3} + 3t(-1)t^{-2} - t^{-1} &\stackrel{?}{=} 0 \\
4t^{-1} - 3t^{-1} - t^{-1} &\stackrel{?}{=} 0 \\
0 &= 0
\end{aligned}$$

The second solution is verified.